MPC-Based Efficient Energy Control and Cost Estimation of HVAC in Buildings

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***Abstract*—The resistor-capacitor network (RC models) is a common approach to model thermal systems in buildings that proves advantageous in improving a building's energy efficiency. This paper presents a framework for energy consumption cost estimation and power efficient control in buildings-based model predictive control. The proposed framework calculates the electricity cost for dwellings based on their sizes using RC models. Additionally, it ensures consistent thermal comfort within the controlled building even during performance issues. The calculation of the cost of energy consumption takes into account the electricity tariff provided by Hydro-Quebec, Montreal, Quebec, Canada. Model Predictive Control (MPC) along with two backup controllers (ON/OFF control and Proportional-Derivative-Integral (PID) control) optimize the thermal model in a building, ensuring the desired indoor temperature efficiently with low cost. The Simulation results conducted on the Matlab/Simulink platform demonstrated that MPC control outperforms the other controllers in terms of energy consumption minimization and cost.**

***Index Terms*—Power consumption cost, model predictive control, smart buildings, thermal systems control.**

I. INTRODUCTION

Approximately 20% to 40% of the existing energy consumption is associated with the building industries, and this proportion is consistently rising between 0.5% to 5% in western countries [1]. The primary consumers of electricity on a global scale are the residential and commercial sectors, collectively accounting for approximately 60% of the world's electricity usage, as reported by the United Nations Environment Program (UNEP) [2]. Consequently, it holds significant economic and environmental importance to devise efficient strategies for reducing power consumption in buildings and reducing the cost accordingly. Such efforts can make a substantial contribution to the smart buildings initiative [7].

A diverse range of traditional and contemporary control methods have been created and applied to regulate power consumption is building systems, with Model Predictive Control (MPC) being a significant technique in this domain [1]–[3], [5], [8], [10]. Despite the presence of certain limitations, such as model dependence, tuning complexities, and real-system implementation challenges, the MPC controller is widely recognized as one of the most potent methods for controlling energy consumption in the realm of smart buildings [1], [2], [9]. The MPC control is a particularly prevalent technique in this domain, largely due to its adeptness in managing constraints, dynamic processes, time variations, delays, uncertainties, and disturbances. The controller also demonstrates the capacity to effectively handle multivariable and nonlinear systems, with accurate predictions and efficient performance [1], [4].

This topic has garnered significant interest, leading to the publication of review papers on the matter. For instance, the MPC control showed superior performance in energy efficiency and comfort criteria when compared to the other controllers [8]. In [9], Various formulations of the MPC controller, including centralized, decentralized, and distributed, along with the proportional-integral and derivative (PID) controller, were implemented on a multi-zone dwelling to achieve temperature tracking and minimize the amount of power used. A new framework of decentralized model predictive control (DMPC) drawing from principles of game theory and discrete event system was implemented to four zone building established that the developed approach validated to significantly reduce the required maximum power while simultaneously ensuring thermal comfort is maintained within an acceptable level [1]. The MPC approach has demonstrated its effectiveness in efficiently managing optimal controls for cold storage [13]. A tube-based robust MPC control was proposed and implemented for indoor temperature control. The outcomes demonstrate that the suggested approach can lead to a substantial reduction in operating expenses, with a minimum of 24% improvement in contrast to classical MPC control schemes. Moreover, it excels in maintaining better control over indoor temperature [14].

In spite of the extensive literature available within the context of smart buildings aimed at optimizing power consumption, as previously indicated, none of these studies have explored the computation of energy expenditures relative to the building sizes based on the RC thermal models. In addition, although the MPC is recognized as a robust and versatile design technique for controlling various physical systems, it can encounter performance obstacles attributed to factors like model mismatch and improper tuning of parameters including prediction horizon, control horizon, and cost function weights [11]. Therefore, this paper presents an straitforward solution in the form of a switching control framework to preempt performance problems stemming from the reasons mentioned above. To recap, this work's principal contributions can be encapsulated in two primary facets:

1) We present a framework designed for facilitating controller switching and estimating energy consumption costs based on the zone's sizes.
2) We introduce a simple solution to mitigate performance problem of the whole system when employing the MPC controller with less complexity.

II. Problem Formulation

The essential control objective is to efficiently regulate the level of the thermal comfort within the building while minimizing power consumption and then calculate the energy consumption cost of the controlled zone, all of this is achieved while maintaining compliance with designer imposed restrictions. This can be expressed as an MPC algorithm involves a quadratic optimization problem.

A. Thermal Dynamics Modeling of Buildings

Within this part, we utilize continuous-time differential equations to demonstrate the thermal model of the system, following the approach outened in [2], [3]. Subsequently, we will discretize the model to employ it for controller implementation. The system model can be expressed as:

$$\frac{\mathrm{d}T_l}{\mathrm{d}t} = \frac{1}{C_l R_l^a}(T_a - T_l) + \frac{1}{C_l}\sum_{j=1,j\neq l}^{F}\frac{1}{R_{jl}^d}(T_j - T_l) + \frac{1}{C_l}\Phi_l, \tag{1}$$

Here, F denotes the zones' numbers. T_l (where $l \in 1,\ldots,F$) represents the temperature inside the Zone l. T_j stands for the internal temperature of an adjacent Zone j ($j \in 1,\ldots,F\backslash l$). T_a represents the external temperature. R_l^a denotes the thermal resistance from Zone l to the ambient outdoor temperature. C_l signifies the heat capacity of Zone l. R_{jl}^d describes the thermal resistance between the neighboring zones l and j. Φ_l represents the power supplied to the thermal device situated in Zone l. It's essential to note that this model is a generic representation widely utilized in prior works. For simplicity's sake, we assume in this work that one thermal appliance is placed in each zone. The equation described in Equation (1) can be depicted through the below state-space model in a continuous-time representation:

$$\begin{aligned}\dot{x} &= Ax + Bu + Wd\\ y &= Cx,\end{aligned} \tag{2}$$

where $u = [u_1,\ldots,u_F]^T$ is the control input sequence, $x = [T_1,\ldots,T_F]^T$ denotes the states of the systems. The system output, demonstrated as $y = [y_1,\ldots,y_F]^T$, which represents the internal temperature in all the zones. Additionally, $d = [T_a^1,\ldots,T_a^F]^T$ symbolizes the disturbance in Zone l. The matrices A, B, and W in Equation (2) are provided below:

$$A = \begin{bmatrix} A_1 & \frac{1}{R_{21}^d C_1} & \cdots & \frac{1}{R_{F1}^d C_1} \\ \frac{1}{R_{12}^d C_2} & A_2 & \cdots & \frac{1}{R_{F2}^d C_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{R_{1F}^d C_N} & \frac{1}{R_{2F}^d C_N} & \cdots & A_F \end{bmatrix},$$

$$B = \mathrm{diag}\begin{bmatrix} B_1 & \cdots & B_F \end{bmatrix}, W = \mathrm{diag}\begin{bmatrix} W_1 & \cdots & W_F \end{bmatrix},$$

with

$$A_l = -\frac{1}{R_l^a C_l} - \frac{1}{C_l}\sum_{j=1,j\neq l}^{F}\frac{1}{R_{jl}^d}, B_l = \frac{1}{C_l},\ W_l = \frac{1}{R_l^a C_l}.$$

In Equation (2), C is a square identity matrix with dimensions F.

The predictive feedback HVAC control signal is determined through the minimization of the performance cost, a function that encompasses both the control input and the system state. The MPC optimization problem's solution is executed in an open-loop fashion. subsequently applying solely the initial element of the generated control input sequences to the controlled plant. This cycle is reiterated periodically, taking into account updated measurements. The below block diagram (Figure 1) depicting the closed-loop functionality of the HVAC MPC controller.

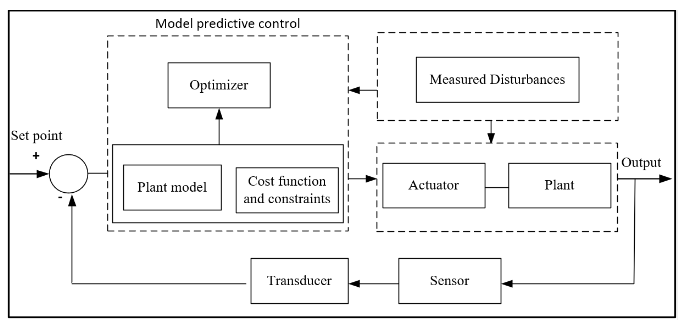

Fig. 1. MPC block diagram.

B. MPC Setup

The discrete-time representation of the model is given by discretizing equation (2) as:

$$\begin{aligned} x(k+1) &= A_{dis}x(k) + B_{dis}u(k) + W_{dis}d(k),\\ y(k) &= C_{dis}x(k), \end{aligned} \tag{3}$$

The state vector is represented by $x(k) \in \mathbb{R}^F$, the control input vector (heating power input) is $u(k) \in \mathbb{R}^F$, and the system output vector is $y(k) \in \mathbb{R}^F$.

If we suppose $x_d(k) \in \mathbb{R}^F$ is the desired state and $er(k) = x(k) - x_d(k)$ represents the error vector. The control input applied to the system is determined by solving the subsequent optimization problem at every time step t:

$$f = \min_{u_i(0)} \sum_{k=0}^{N-1} er(k)^T Q er(k) + u^T(k) R u(k) \tag{4a}$$

$$\text{s.t} \quad x(k+1) = A_{dis}x(k) + B_{dis}u(k) + W_{dis}d(k),$$

$$x_0 = x(t), \tag{4b}$$

$$x_{\min} \leq x(k) \leq x_{\max}, \tag{4c}$$

$$0 \leq u(k) \leq u_{\max}, \tag{4d}$$

for $k = 0, \ldots, N$, where N represents the prediction horizon that is set by the designer. Within the cost function detailed in (4), $R = R^T > 0$ and $Q = Q^T \geq 0$ serves as weighting matrices to impose penalties on HVAC control input and the deviations in tracking, correspondingly.

The solution of the equation (4) yields a set of control values $U^*(x(t)) = \{u_0^*, \ldots, u_N^*\}$. Among these, solely the initial element $u(t) = u_0^*$ will be chosen and sent to the process. A solver is needed to provide the solution of the MPC control problem at every sampling moment. Thus, the open source YALMIP toolbox [6] will be used with the MPC control design to fulfill this requirement.

III. Proposed Solution

Reiterating what was mentioned in Section (I), the aim is to regulate and adjust the thermal comfort level while minimizing the power consumption. Moreover, empower users to compute electricity costs considering the controlled zone's dimensions. To ensure that customers receive a good service resulting in the desired indoor temperature at their specified preferences, along with minimal power consumption and cost, it's imperative to maintain continuous efficiency all the time. To overcome challenges associated with the MPC controller, such as issues with modeling consistency and complexities in tuning MPC control parameters (like prediction horizon, control horizon, and cost function weighting), we've introduced a graphical user interface (GUI) tool. This tool has been designed to effectively address the mentioned MPC controller issues. The introduced framework facilitates seamless transitions between three specific control strategies, with MPC acting as the primary controller and PID and ON/OFF controllers serving as backup alternatives. The MPC control takes the lead within this interface, managing thermal comfort and optimizing power consumption. Should any complications arise, users have the option to switch to two alternative backup controllers ensuring efficient regulation of system performance. Once the issue is resolved, seamlessly reverting to the primary control is executed.

Note that many approaches in literature have been implemented to guarantee the stability performance of the MPC controller such as Lyapunov criteria [1], [12]. However, that requires an additional computations and considerations during the design phase to ensure the stability which increase the control complexity [2], [11] which restricts the application on the real systems. In particular. when the embedded hardware units have limited computational resources.

The proposed tool is equipped with the capability to enable users to estimate their energy consumption costs based on the zone sizes. This estimation is derived from the electricity pricing structure provided by Hydro-Quebec, a significant power company operating in Montreal, Quebec, Canada.

IV. Simulation Experiments

In this paper, a framework is proposed and created by using the graphical user interface (GUI) to calculate the energy consumption cost of the selected zones based on the tariff of Hydro-Quebec Company in Montreal-Quebec, Canada. All simulations are conducted using the Matlab/Simulink platform.

The objective is to assure that the output (indoor temperature) closely tracks the pre-specified set-point of 22°C and determined by the user. The fluctuation in the outdoor temperature is illustrated in Figure 2.

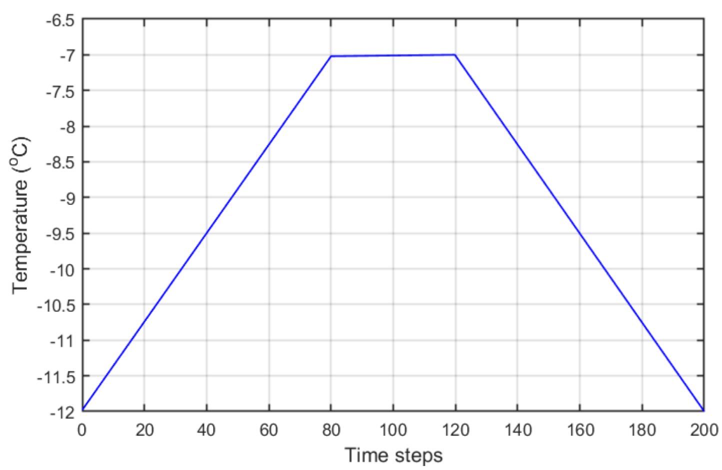

Fig. 2. Disturbance of ambient temperature.

In this simulation experiment, the time scale is standardized to 200 time units. The prediction horizon is chosen as N = 10, the sampling period is Ts= 3-time steps and the initial indoor temperature is 0°C. The simulation corresponds to approximately 10 hours in real-time.

The RC parameters of the thermal model for these specific zones were sourced from [2], are outlined in Table I and Table II. The control approaches were implemented on two distinct single zones varying in sizes (zone 1 and zone 3) and outfitted with a heater each.

TABLE I
Configuration of thermal parameters for heaters.

Zone	1	2	3	4
R_j^a	69.079	88.652	128.205	105.412
C_j	0.94	0.94	0.78	0.78

The thermal system's output in Zone 1 and the corresponding control signals for each controller are depicted in Figure 3 and Figure 4, respectively.

TABLE II
PARAMETERS OF THERMAL RESISTANCES FOR HEATERS.

R^r_{12}, R^r_{21}	R^r_{13}, R^r_{31}	R^r_{14}, R^r_{41}	R^r_{23}, R^r_{32}	R^r_{24}, R^r_{42}
709.2	1063.8	1063.8	1063.8	1063.8

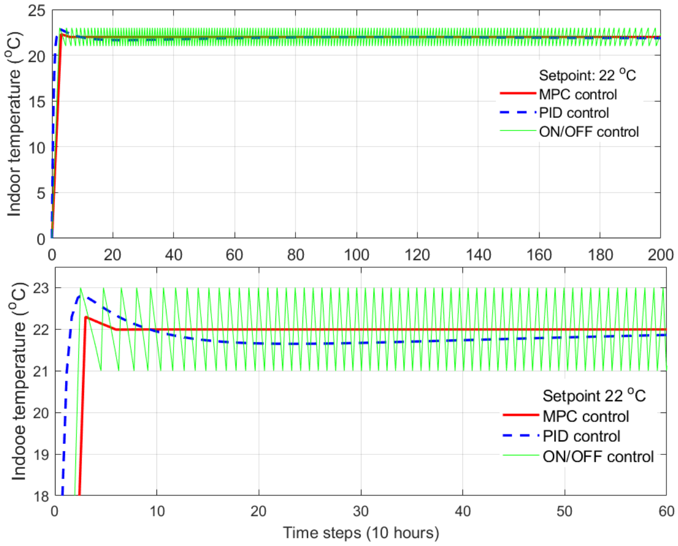

Fig. 3. Indoor Temperature of Zone 1.

We repeated the experiment for Zone 3 with the same setup, and the resulting indoor temperature data is presented in Figure 5. Simultaneously, Figure 6 illustrates the control signals generated by the applied controllers.

The simulation results for both zones make it evident that MPC excels in achieving the desired comfort level within the controlled zone with notably lower power consumption. Furthermore, the output response of both thermal systems for the two zones closely follows the setpoint throughout

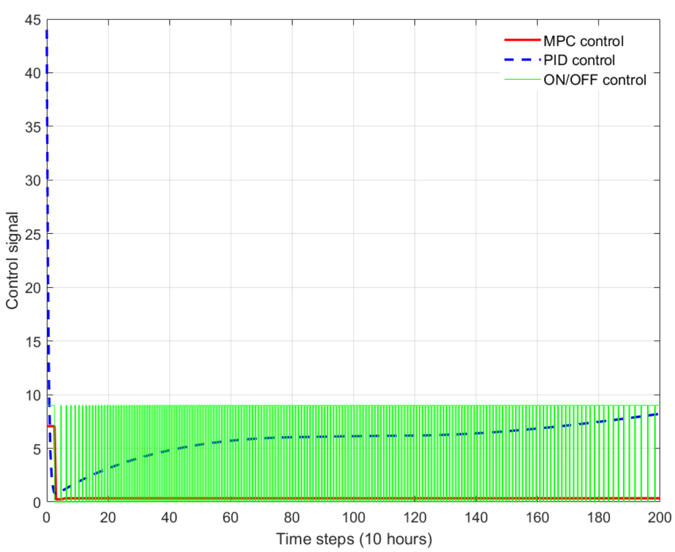

Fig. 4. Control signal of the proposed controllers for zone 1

the simulation duration when we use the MPC. However, when utilizing PID control and ON/OFF controllers, there is a slight deviation observed around the setpoint. The time domain specifications of the system output are illustrated in Table III, t is obvious that even-though the both backup controllers casue faster output response as they have less rise time, the output controlled by the MPC control has less overshoot and less settling time.

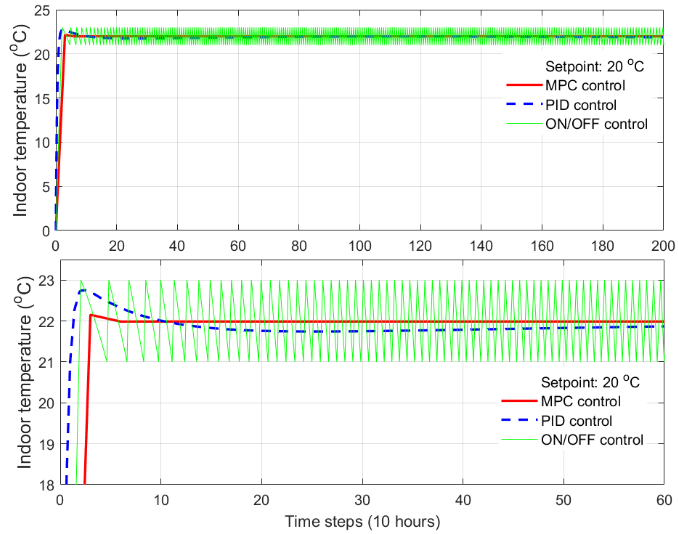

Fig. 5. Indoor Temperature of Zone 3.

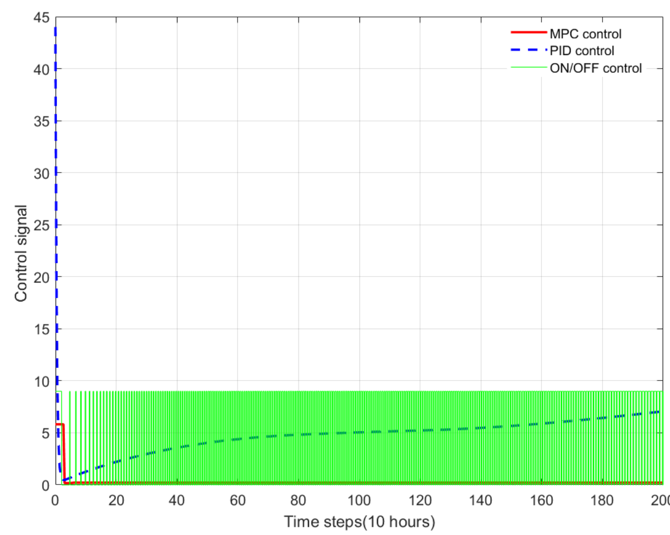

Fig. 6. Control signal of the proposed controllers for zone 3

To determine the energy consumption needed by each controller in Zone 1, we utilized a methodology that involves integrating the power signal of each controller over the simulation duration. Fig. 7 presents the data, indicating that MPC control requires a total energy consumption of 0.522 kWh. In contrast, users that use the PID control and the ON/OFF control consume 7.31 kWh and 6.678 kWh, respectively.

TABLE III
TIME DOMAIN SPECIFICATIONS OF THE THERMAL SYSTEM OUTPUT FOR ZONE 1 WITH THREE CONTROLLERS.

	Rise time(sec)	Overshoot (%)	Settling time(sec)
MPC	2.3667	1.3761	2.8979
PID	0.9281	4.2869	6.2066
ON/OFF	1.9538	2.7832	199.4694

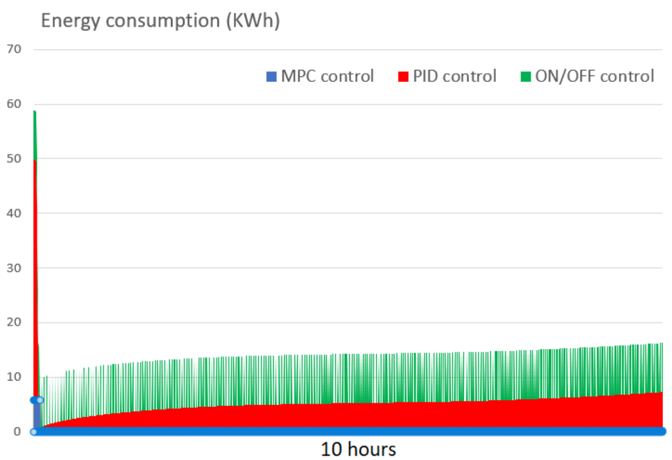

Fig. 7. Energy consumption

For cost estimation based on Hydro-Quebec regulations in Montreal, QC, Canada, (4.678¢/kWh) the corresponding amounts from Fig. 7 are what clients would need to pay over the specified duration. Clearly, MPC proves to be superior in both energy efficiency and performance compared to the backup controllers. Utilizing MPC could lead to substantial cost savings, emphasizing its potential for significant financial benefits. The GUI proposed tool as shown in Fig. 8 gives the customer a choice to estimate the cost of energy consumption based on the area of the zone.

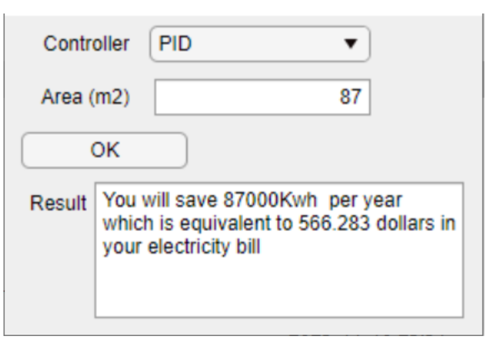

Fig. 8. Switching control window

V. CONCLUSION

This paper introduces a framework for estimating energy consumption costs and implementing power-efficient control in buildings through model predictive control. The framework uses RC models to calculate electricity costs based on building sizes, ensuring consistent thermal comfort even during performance issues. The energy consumption cost estimation considers the electricity tariff from Hydro-Quebec, Canada. The study employs MPC control alongside two backup controllers (ON/OFF and PID) to regulate the thermal models of the building within a switching control window, maintaining indoor temperature as per user-defined settings with reduced power consumption and cost. Simulation results using Matlab/Simulink demonstrate MPC's superior performance in minimizing energy consumption and associated costs compared to the other controllers.

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